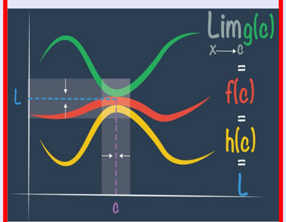
Math 261

Spring 2022

Lecture 1



Math 261

15 Weeks

M W

8:50 - 11:20

You must have

1) Internet access with printer.

2) Access to Canvas Download Canvas App for Student

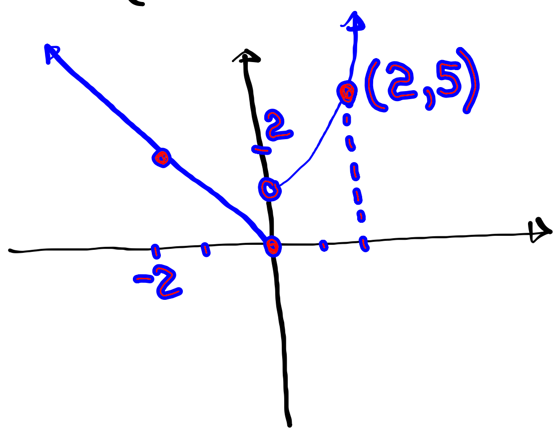
3) TI 83 or TI 84 calculator with you in class at all time.

4) Final Exam: June 6, 2022

February 7, 2022

Review:

$$f(x) = \begin{cases} |x| & \text{if } x \le 0 \\ x^2 + 1 & \text{if } x > 0 \end{cases}$$



Piece-wise Function

1) Find $f(-2)$

$f(-2) = |-2| = 2$

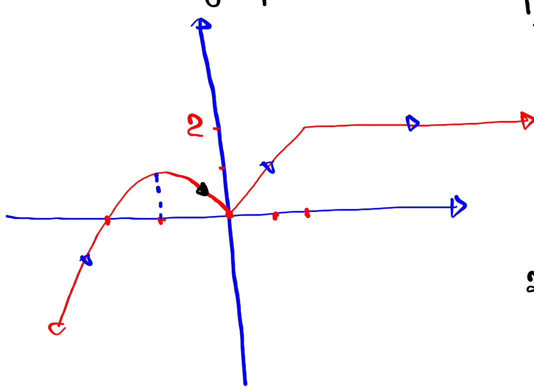
2) Find $f(0)$

$f(0) = |0| = 0$

3) Find $f(2)$

$f(2) = (2)^2 + 1 = 5$

Consider the graph below



1) This graph belongs to a function. How?

Try vertical line test.

2) Domain $(-\infty, \infty)$

Range $(-\infty, 2]$

3) Increasing $(-\infty, -1) \cup (0, 2)$

Decreasing $(-1, 0)$ Constant $(2, \infty)$

4) Intercepts: x-Int $(-2, 0), (0, 0)$

y-Int $(0, 0)$

Consider the Function $f(x) = \underbrace{x^2 - 4x + 4}$

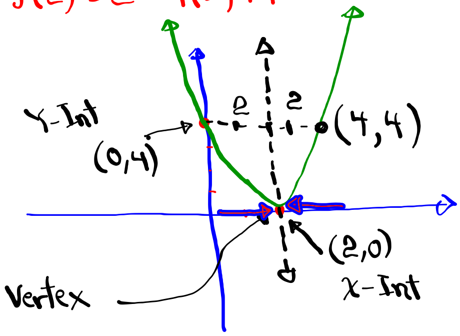
Quadratic Function $\to f(x) = ax^2 + bx + c$

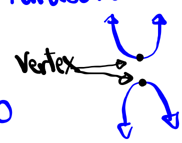
$f(0) = 0^2 - 4(0) + 4 = 4$

$f(2) = 2^2 - 4(2) + 4 = 0$

Name of graph

Parabola





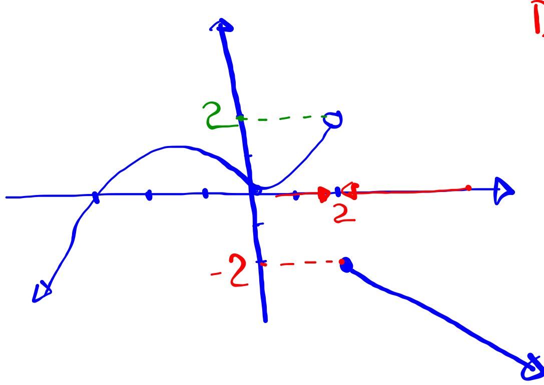
$a \neq 0$

as $x \to 2$ from the right $\Rightarrow f(x) \to 0$

as $x \to 2$ from the left $\Rightarrow f(x) \to 0$

Introduction to limits

Consider the graph below



Domain $(-\infty, \infty)$

Range $(-\infty, 2)$

All intercepts:

x-Ints: $(-3,0), (0,0)$

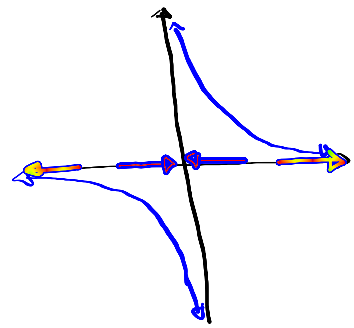
Y-Int: $(0,0)$

as $x \to 2$ from the right, $f(x) \to -2$

as $x \to 2$ " " left, $f(x) \to 2$

$f(x) = \frac{1}{x}$

Reciprocal Function



Domain $(-\infty,0) \cup (0,\infty)$

Range $(-\infty,0) \cup (0,\infty)$

All intercepts None

as $x \to 0$ from right $\Rightarrow f(x) \to \infty$

as $x \to 0$ 〃 left $\Rightarrow f(x) \to -\infty$

as $x \to \infty \Rightarrow f(x) \to 0$

as $x \to -\infty \Rightarrow f(x) \to 0$

Given $f(x) = \frac{2x^2 - x - 1}{x-1}$

1) Domain

Deno. $\neq 0$

$x - 1 \neq 0 \Rightarrow (-\infty,1) \cup (1,\infty)$

$x \neq 1$

2) Simplify $f(x)$

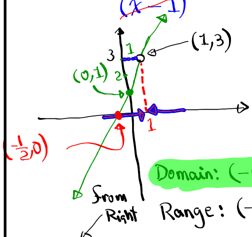
$f(x) = \frac{\cancel{(x-1)}(2x+1)}{\cancel{(x-1)}}$

$\underbrace{f(x) = 2x + 1}_{\text{Linear Function}}$, $x \neq 1$

$f(x) = mx + b$

Slope $m = 2 = \frac{2}{1}$

Y-Int $(0,1)$



Domain: $(-\infty,1) \cup (1,\infty)$

Range: $(-\infty,3) \cup (3,\infty)$

$x \to 1^+ \Rightarrow f(x) \to 3$

$x \to 1^- \Rightarrow f(x) \to 3$

From left

$\Rightarrow$ what is $f(1)$? undefined

not in the domain of $f(x)$

Find x-Int:

x-Int $\Rightarrow y = 0 \Rightarrow f(x) = 0 \Rightarrow$ Numerator $= 0$

$2x^2 - x - 1 = 0$

$(x-1)(2x+1) = 0$

~~$x - 1 = 0$~~

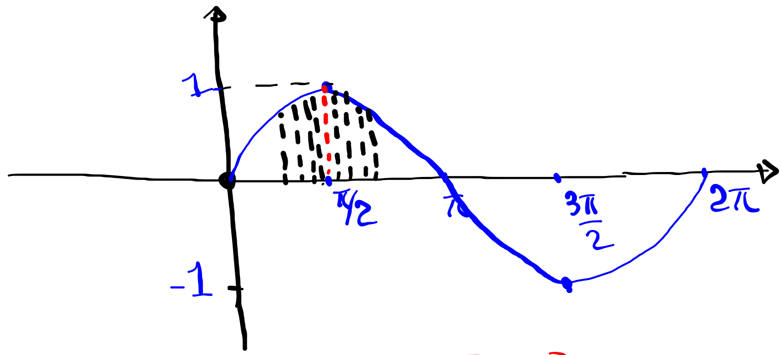
~~$x = 1$~~

$2x + 1 = 0$

$x = \frac{-1}{2}$

x-Int $(\frac{-1}{2}, 0)$

$f(x) = \sin x \quad , \quad 0 \le x \le 2\pi$



x	$f(x)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

Domain $[0, 2\pi]$

Range $[-1, 1]$

Intercepts

x-Int: $(0,0), (\pi,0), (2\pi,0)$

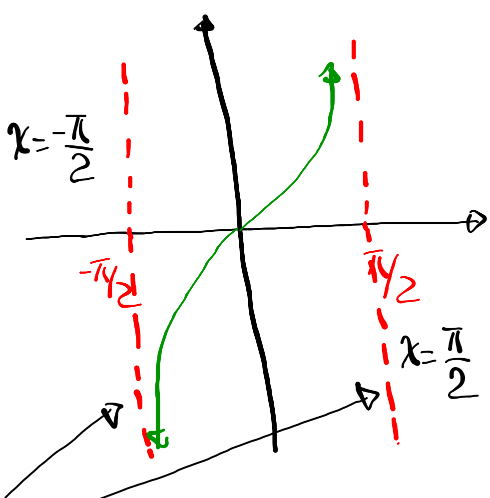
Y-Int $(0,0)$

as $x \to \frac{\pi}{2}^{+} \Rightarrow f(x) \to 1$

as $x \to \frac{\pi}{2}^{-} \Rightarrow f(x) \to 1$

$f(x) = \tan x \,, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

1) Graph



Vertical Asymptotes

2) Domain

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3) Range

$(-\infty, \infty) = \mathbb{R}$

4) Intercepts

$(0,0)$

February 7, 2022

Given $f(x) = ax^2 + c$

Find the difference quotient.

$$\underbrace{\frac{f(x+h) - f(x)}{h}}$$

$$f(x+h) = a(x+h)^2 + c = a(x^2 + 2hx + h^2) + c$$
$$= ax^2 + 2ahx + ah^2 + c$$

$$f(x+h) - f(x) = \cancel{ax^2} + 2ahx + ah^2 \cancel{+c} \cancel{-ax^2} \cancel{-c}$$

$$= 2ahx + ah^2$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2ahx + ah^2}{h} = \frac{\cancel{h}(2ax + ah)}{\cancel{h}}$$

$$= \boxed{2ax + ah}$$

Given $f(x) = x^3$, find the difference quotient, then simplify, and evaluate for $h=0$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{(x+h)(x+h)^2 - x^3}{h}$$

$$= \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h}$$

$$= \frac{x^3 + 2x^2h + xh^2 + hx^2 + 2xh^2 + h^3 - x^3}{h}$$

$$= \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x^3}}{h} = \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

Let $h=0$ $\leftarrow$ $= 3x^2 + 3xh + h^2$

$$3x^2 + 3x(0) + 0^2$$
$$= \boxed{3x^2}$$

Graph $|x| - |y| = 4$

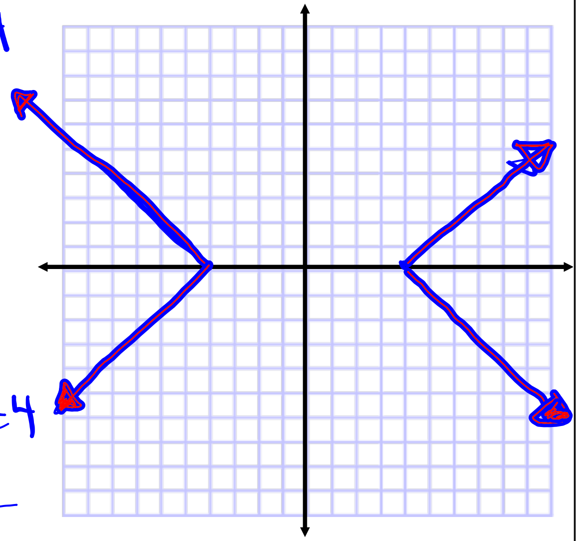
QI

$x>0,\ y>0$

$|x| = x,\ |y| = y$

$x - y = 4$

x	y
0	-4
4	0



QII

$x<0\ ,\ y>0$

$|x| = -x \qquad |y| = y$

$-x - y = 4$

x	y
0	-4
-4	0

QIII

$x<0\ ,\ y<0$

$|x| = -x \qquad |y| = -y$

$-x - (-y) = 4$

$-x + y = 4$

x	y
0	4
-4	0

Simplify

$$(1 + \tan x)^2 - \sec^2 x - \frac{1}{\cot x}$$

$(A+B)^2 = A^2 + 2AB + B^2$

$$= 1 + 2\tan x + \tan^2 x - \sec^2 x - \frac{1}{\cot x}$$

$$= \cancel{\sec^2 x} + \underline{2\tan x} - \cancel{\sec^2 x} - \underline{\tan x}$$

$$= \boxed{\tan x}$$

Graph $x^2 + y^2 = 16 \longrightarrow (x-h)^2 + (y-k)^2 = r^2$

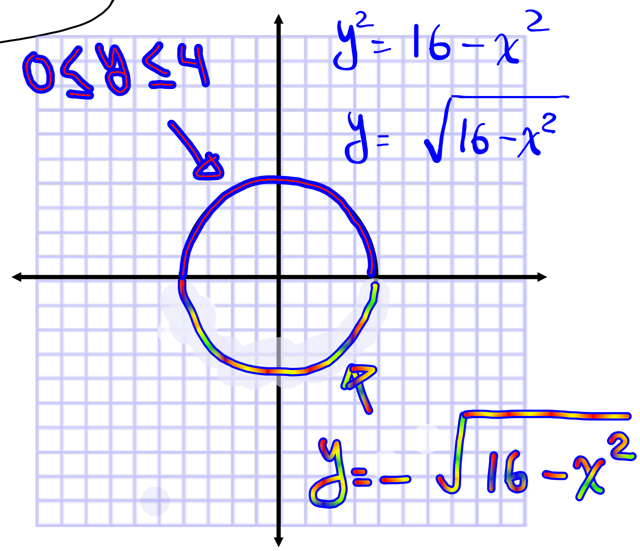
Circle

Center $(0,0)$

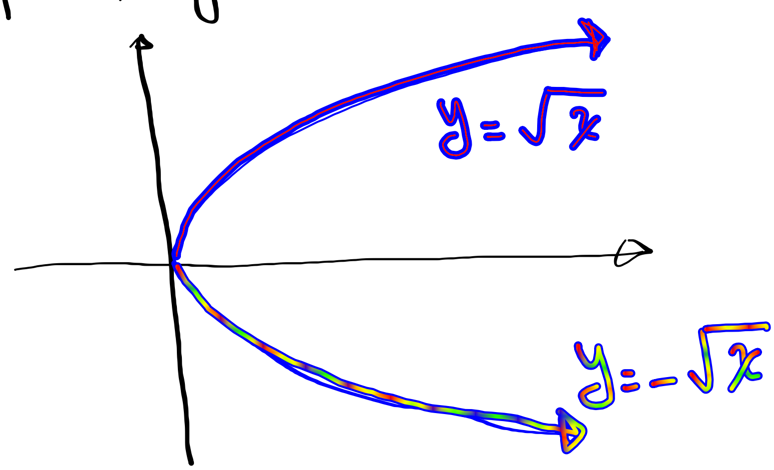
Radius 4

Domain: $[-4,4]$

Range: $[-4,4]$



Graph $x = y^2$



Class QZ 1:

Use quadratic formula to Solve $3x^2-5x+2=0$.

Final Ans in a Solution Set.

$ax^2+bx+c=0$

$a=3$

$b=-5$

$c=2$

$b^2-4ac=(-5)^2-4(3)(2)=25-24=\boxed{1}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-5)\pm\sqrt{1}}{2(3)}=\frac{5\pm1}{6}$$

$$x=\frac{5+1}{6}=\frac{6}{6}=1$$

$$x=\frac{5-1}{6}=\frac{4}{6}=\frac{2}{3}$$

$\left\{\frac{2}{3}, 1\right\}$